Enrollment No: _______________________ Exam Seat No: _______________________

C. U. SHAH UNIVERSITY

Summer Examination-2020

Subject Name: Mathematics-I

Subject Code: 4SC01MAT1 **Branch: B.Sc. (All)**

Semester : 1 **Date : 28/02/2020** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) State Rolle's theorem. 02

b) Find Maclaurin's series of $f(x) = e^x$. 02

c) Define: Scalar matrix. 01

d) Write down necessary and sufficient condition for the differential equation to be exact. 01

e) $(x^2 + y^2)dx - 2xydy = 0$ is differential equation of type_____ 01

(a) Homogeneous (b) Bernoulli's (c) Exact (d)Linear

f) Write form of Clairaut's equation. 01

g) $\lim_{x\to 0} \frac{\tan x}{x}$ is of the form 01

(a) $\frac{\infty}{\infty}$ (b) $\infty - \infty$ (c) $\frac{0}{0}$ (d) 0^0

h) Find the order of differential equation 01

$$\frac{d^2y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}.$$

i) State Caley-Hamilton theorem 01

j) The integrating factor of the differential equation $ydx - xdy = 0$ is 01

(a) $\frac{1}{x}$ (b) $\frac{1}{y^2}$ (c) $\frac{1}{y}$ (d) None of these

k) Polar form of equation $x^2 + y^2 = 6xy$ is 01

(a) $1 = 3\sin 2\theta$ (b) $1 = 2sin3\theta$ (c) $1 = 3cos2\theta$(d) None

l) Write down Maclurin's series of $\cos x$. 01

Attempt any four questions from Q-2 to Q-8

Q-2		**Attempt all questions**	**(14)**
	a)	State and Leibnitz's theorem.	07
	b)	Evaluate $\lim_{x \to \frac{\pi}{2}} (sin\ x)^{tan\ x}$.	05
	c)	State Langrage's mean value theorem.	02
Q-3		**Attempt all questions**	**(14)**
	a)	Verify Lagrange's mean value theorem for the function, $f(x) = (x-1)(x-2)(x-3), \forall x \in [1,4]$.	06
	b)	Solve: $\frac{dy}{dx} - \frac{dx}{dy} = \frac{x}{y} - \frac{y}{x}$	06
	c)	State Taylor's series	02
Q-4		**Attempt all questions**	**(14)**
	a)	If $y = (\sin^{-1} x)^2$, show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n$, hence find $(y_n)_0$.	06
	b)	For $x > 0$, show that $\frac{x}{1+x^2} < \tan^{-1} x < x$.	05
	c)	Evaluate$\lim_{x \to 0} \frac{xcosx - \log(1+x)}{x^2}$.	03
Q-5		**Attempt all questions**	**(14)**
	a)	If $(2, \frac{\pi}{4}, \frac{\pi}{6})$ are spherical co-ordinates for a point then find cartesian and cylindrical co-ordinates.	06
	b)	Verify Cauchy's mean value theorem for the function $f(x) = \sin x\ , g(x) = \cos x\ , \forall x \in \left[0, \frac{\pi}{2}\right]$.	05
	c)	Convert the equation $x^3 = y^2(2a - x)$to the polar equation.	03
Q-6		**Attempt all questions**	**(14)**
	a)	Reduce the matrix $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ into its normal form and hence find its rank.	06
	b)	Find the A^{-1} of $A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$ by Gauss-Jordan method	06
	c)	Evaluate: $\begin{vmatrix} 2 & -1 & 1 \\ 3 & -2 & 1 \\ 9 & -5 & 4 \end{vmatrix}$	02
Q-7		**Attempt all questions**	**(14)**
	a)	Verify Caley-Hamilton theorem for the matrix	07

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

b) Solve given system of equation by Gauss elimination method 05

$$x + y + 2z = 9$$
$$2x + 4y - 3z = 1$$
$$3x + 6y - 5z = 0$$

c) Find the equation of sphere for which $A(2, -3, 4)$ and $B(-2, 3, -4)$ are the extremities of a diameter. 02

Q-8 Attempt all questions (14)

a) Find eigenvalues and eigenvectors of the matrix 05

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}.$$

b) Solve: $\frac{dy}{dx} = x^3 - 2xy$. 05

c) Find the equation of the sphere which touches the sphere $x^2 + y^2 + z^2 - x + 3y + 2z - 3 = 0$, at the point $(1, 1, -1)$ and passes through the origin. 04